

## Precalc Warm Up # 11-5

Evaluate:

1.  $\sum_{k=4}^{25} (6 - 2k)$

2.  $\sum_{t=1}^8 (-2)^t$

3.  $68 + 27.2 + 10.88 + \dots$

1. 16, 15.5, 15, 14.5, ...

a) Find the 20<sup>th</sup> term.

b) Which term is equal to 0?

c) The sum of the first n terms is 246. Find the possible values for n.

a)  $d = -0.5$   $a_{20} = 16 - 0.5(19)$   
 $a_{20} = 6.5$

b)  $0 = 16 - 0.5(n-1)$   
 $-16 = -0.5(n-1)$   
 $32 = n-1$   
 $n = 33$

c)  $246 = \frac{n}{2} [2(16) - 0.5(n-1)]$   
 $-2(492 = 32n - 0.5n^2 + 0.5n)$   
 $0 = n^2 - 65n + 984$   
 $0 = (n-24)(n-41)$   
 $n = 24, 41$

2. The 5<sup>th</sup> term of a geometric series is 12 and the 7<sup>th</sup> term is 3. Find the 2 possible values of the sum to infinity of the series.

$g_5 = 12$   $g_7 = 3$

$12r^2 = 3$   
 $r^2 = \frac{1}{4}$

$r = \pm \frac{1}{2}$

$12 = g_1 \left(\pm \frac{1}{2}\right)^4$   
 $12 = g_1 \left(\frac{1}{16}\right)$   
 $g_1 = 192$

$S_{\infty} = \frac{192}{1 - \frac{1}{2}}$

$S_{\infty} = 384$

$S_{\infty} = \frac{192}{1 + \frac{1}{2}}$

$= \frac{192}{\frac{3}{2}}$

$= 192 \left(\frac{2}{3}\right)$

$S_{\infty} = 128$

3. The 5<sup>th</sup> term of a geometric sequence is 128 and the 6<sup>th</sup> term is 512.

a) Find the common ratio and the first term.

b) Which term has a value of 32,768?

c) How many terms are needed before the sum of all the terms in the sequence exceeds 100,000?

$$\begin{aligned}
 &g_5 = 128 \quad g_6 = 512 \\
 \text{a) } &r = \frac{512}{128} \\
 &\boxed{r = 4} \\
 &128 = g_1(4)^4 \\
 &\boxed{g_1 = \frac{1}{2}} \\
 \text{b) } &32,768 = \frac{1}{2}(4)^{n-1} \\
 &65536 = 4^{n-1} \\
 &4^8 = 4^{n-1} \\
 &\boxed{n = 9} \\
 \text{c) } &S_n > 100,000 \\
 &\frac{0.5(1-4^n)}{1-4} > 100,000 \\
 &0.5(1-4^n) < -300,000 \\
 &1-4^n < -600,000 \\
 &4^n > 600,001 \\
 &n > \frac{\ln 600,001}{\ln 4} \\
 &n > \approx 9.60 \\
 &\boxed{10 \text{ terms}}
 \end{aligned}$$

4.

one million dollars:

$$\begin{aligned}
 \text{a) } &y = 25,000(1.03)^{10} \\
 &\boxed{y \approx \$33,597.91}
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } &S_{35} = \frac{25750(1-(1.03)^{35})}{1-1.03} \\
 &\boxed{S_{35} \approx \$1,556,898.61}
 \end{aligned}$$

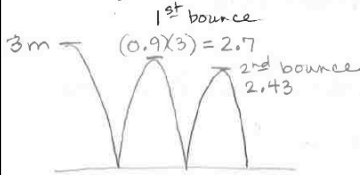
$$\begin{aligned}
 \text{c) } &g_n > 35,000 \\
 &25750(1.03)^{n-1} > 35,000 \\
 &(1.03)^{n-1} > \frac{140}{103}
 \end{aligned}$$

$$\begin{aligned}
 &(n-1)\ln(1.03) > \ln\left(\frac{140}{103}\right) \\
 &n > \approx 11.4 \quad \leftarrow n > \frac{\ln\left(\frac{140}{103}\right)}{\ln 1.03} + 1 \\
 &\boxed{12 \text{ years}}
 \end{aligned}$$

$$\begin{aligned}
 \text{d) } &S_n = 1,000,000 \\
 &\frac{25750(1-(1.03)^n)}{1-1.03} = 1 \text{ mil} \\
 &1-(1.03)^n = -\frac{120}{103} \\
 &1.03^n = \frac{223}{103} \\
 &n = \frac{\ln\left(\frac{223}{103}\right)}{\ln 1.03} \\
 &n \approx 26.1 \\
 &\boxed{27 \text{ years}}
 \end{aligned}$$

5. A ball is dropped from a height of 3 m. Each time it hits the ground it bounces up to 90% of its previous height.

- a) How high does it reach after the 5<sup>th</sup> bounce?  
 b) On which bounce does the ball first reach a maximum height of less than 1 m?



$$\begin{aligned} a) \quad g_5 &= 2.7(0.9)^4 \\ &= 1.77147 \\ &\approx 1.77 \text{ m} \end{aligned}$$

$$b) \quad g_n < 1$$

$$2.7(0.9)^{n-1} < 1$$

$$n > \approx 10.4$$

11<sup>th</sup> bounce

\*  $\ln 0.9$  is negative

$$\begin{aligned} (n-1) \ln(0.9) &< \ln\left(\frac{1}{2.7}\right) \\ n &> \frac{\ln\left(\frac{1}{2.7}\right)}{\ln(0.9)} + 1 \end{aligned}$$

SL Book

HW: p. 200 #1-7 LC